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A STUDY ON STOCHASTIC APPLICATION IN LINEAR ALGEBRA

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Abstract

To mention a few fields, stochastic processes are applied in demographics, health care systems, polymer science, physics, telecommunication networks, economics, finance, marketing, and social networks. Each of these fields has a substantial body of literature. Queueing theory, inventory systems, supply chains, manufacturing, healthcare systems, computer and communication networks, dependability, warranty management, mathematical finance, and statistics are some of the areas where we typically find applications. In the sample below, we demonstrate a couple of these applications. Despite the fact that the majority of applications appear to involve continuous time stochastic processes, these applications can easily be changed to discrete time stochastic processes by assuming that the system in question is observed at a discrete set of points, such as once every hour, once per day, etc.

Keywords: Stochastic, Linear, Characteristic.

Introduction

In practically every area of mathematics, linear algebraic systems are crucial. In the case of table-top numerical experiments, such systems may very well be resolved using well-known techniques like LU decomposition, but in a large range of situations, the size of the matrices would make them unstorable even given the ample storage space currently available. However, the matrices in these situations typically have a highly sparse structure, greatly lowering their memory footprint. The disadvantage is that an inverse of a sparse matrix is typically dense, making LU decomposition an impractical method. In these situations, iterative methods are used to solve the linear system by virtue of a fixed-point iteration, such as generalised minimal residues iteration and bi-conjugate gradient stabilised iteration.

Since there is no magic answer to a problem of this complexity, iterative approaches may experience sluggish convergence. Preconditioners, which are matrices that alter the issue and, theoretically, cut down on the amount of iterations and runtime required to solve it, are used to try and mitigate this difficulty. Here, we evaluate the suitability of two approaches for computing mentioned preconditioners: stochastic gradient descent with missing values and stochastic process.

In an algebraic equation known as a linear equation, each term is either a constant or the result of multiplying a constant by one variable (to the first power). One or more variables may be present in a linear equation. In most branches of mathematics, and particularly in applied mathematics, linear equations are ubiquitous. They are highly useful since many nonlinear equations may be converted to linear equations by assuming that the quantities of interest only slightly differ from some "background" condition, despite the fact that they naturally exist when modelling many processes. Exponents are not included in linear equations. This study also examines the situation of an equation for which genuine solutions are sought. In general, all of its information is applicable to linear equations with coefficients and solutions in any field, as well as complex solutions.

Review of Literature

R.G. Ghanem and M.F. Pellissetti (2000) [1], The modification of current iterative solution techniques is driven by the specific characteristics of systems of linear equations that emerge in the setting of the Stochastic Finite Element Method (SFEM). The implementation covered in this work intended to improve preconditioning techniques, MAT-VEC procedures, and data management. It turns out that SFEM-systems require far less effort to solve than their size would imply. The primary concept is based on the observation that the whole system matrix is composed of a small number of sparse submatrices with similar dimensions. Due to this, matrix-vector multiplications at the submatrix level can be carried out very well without assembling the entire coefficient matrix.

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CF Li and others (2006) [2], The stochastic system of linear algebraic equations $(\alpha_1\boldsymbol{A}_1 + \alpha_2\boldsymbol{A}_2 + \cdots + \alpha_m\boldsymbol{A}_m)\boldsymbol{x} = \boldsymbol{b}$ arising from stochastic finite element modelling in computational mechanics is solved using a novel approach in this paper. In this system, $\alpha_i\,(i = 1, \ldots, m)$ denote random variables, $\boldsymbol{A}_i\,(i = 1, \ldots, m)$ real symmetric deterministic matrices, b is a determent, The stochastic equation system is first decoupled by concurrently diagonalizing all of the Ai matrices using a similarity transformation, and the explicit solution is then easily obtained by inverting the sum of the diagonalized stochastic matrices. The joint diagonalization can only be accomplished roughly if all of the matrices Ai don't have the exact same eigen-structure. As a result, the solution is approximative and matches a specific average eigen-structure of the matrix family. In more detail, the traditional Jacobi algorithm for computing a single matrix's eigenvalues is changed to accommodate multiple matrices, and the resulting Jacobi-like joint diagonalization algorithm maintains the core characteristics of the original version, including convergence and an explicit solution for the ideal Givens rotation angle. Three numerical examples are given to show how the approach performs. The following text uses bold capital letters, bold lower case letters, and regular characters to indicate matrices, vectors, and scalars, respectively. A vector (such as x), without further explanation, always denotes a column vector. A matrix or vector's transposition is indicated by the superscript T. A matrix A's entry in the ith row and jth column is denoted by the symbol (A)ij.

The standard form of the linear finite element equations is as follows: The physical meanings of A, x, and b are related to the system under investigation in the equation Ax=b. In the case of a steady-state heat conduction problem, they stand for the heat conduction matrix, the unknown nodal temperature vector, and the nodal temperature-load vector, respectively. For instance, in a straightforward elastostatics problem, A denotes the elastic stiffness matrix, x the unknown nodal displacement vector, and b the nodal load vector. In practise, the proposed deterministic physical model is rarely accurate due to different uncertainties. As a result, a suitable safety factor is frequently used to modify the outcome for a real-world engineering problem. The effectiveness of this deterministic technique has been demonstrated by numerous successful applications. On the other hand, there are a variety of challenges for which deterministic models have been unable to provide appropriate answers, such as issues with rocks, soil, and ground water where stochastic uncertainty takes over. As a result, to describe these complex random physical systems, a more generalised solution technique-the stochastic finite element method (SFEM)-is required. The SFEM is still in its infancy, and there are many fundamental questions that remain unanswered, in contrast to the success of the deterministic FEM. The formulation of the SFEM has recently drawn significant interest from the computational mechanics community due to the ongoing exponential increase in computer power and data storage, and generally speaking, for many engineering applications, the final linear SFEM equations take the following form: The deterministic n x n real symmetric matrices $\boldsymbol{A}_i\,(i = 1, \ldots, m)$, (α1A1+α2A2+⋯+αmAm)x = b Real scalars $\alpha_i\,(i = 1, \ldots, m)$ describe a series of dimensionless random elements that encapsulate various intrinsic uncertainties in the system. Unknown real random vector x and deterministic real vector b have roughly the same physical meanings as their primary counterparts in equation. Equation provides the benefit of decoupling the physical problem with a stochastic character into deterministic and stochastic pieces in a straightforward regular way.

Characterize A Stochastic Process

We analyse stochastic processes and thoroughly research a number of unique kinds of stochastic processes. To accomplish this, we need a method that can unambiguously characterise stochastic processes using mathematics. It is appropriate to start by examining how one "describes" a single random variable since a stochastic process is a collection of random variables.

We show from basic probability theory (see Appendices A and B) that the cumulative distribution function of a single random variable X describes it entirely.

$$F(x) = P(X \leq x), -\infty < x < \infty \qquad (1.1)$$

This is noted that multivariate random type variable (X1, X2, ⋯ , Xn) is being fully expressed through its joint considering cdf

$$F(x1, x2, \cdots, xn) = P(X1 \leq x1, X2 \leq x2, \cdots, Xn \leq xn), \qquad (1.2)$$

for all $-\infty < xi < \infty$ and i = 1, 2, ⋯, n. In continuation, as parameter set like T is considering as finite, process regarding stochastic $\{X(\tau), \tau \in T\}$ is being multivariate random type variable, & therefore is fully expressed through joint cdf. As what about related to case when T is being not finite?

Assume case T = {0, 1, 2, · · ·} first. One may naively try to find the joint cdf's direct extension to the infinite dimensional case by doing the following:

$F(x_0, x_1, \cdots) = P(X_0 \leq x_0, X_1 \leq x_1, \cdots), -\infty < x_i < \infty, i = 0, 1, \cdots .$ (1.3)

However, the probability on the right-hand side of the aforementioned equation is usually zero or one. Consequently, it is unlikely that such a function will provide much insight into the stochastic process $\{X_n, n \geq 0\}$.

Therefore, we must search for a substitute. Imagine receiving a family of joint cdfs with finite dimensions as $\{F_n, n \geq 0\}$ such that

$F_n(x_0, x_1, \cdots , x_n) = P(X_0 \leq x_0, X_1 \leq x_1, \cdots , X_n \leq x_n),$ (1.4)

for all $-\infty < x_i < \infty$, and $i = 0, 1, \cdots , n$. If such a family fulfils, it is referred to as consistent

$F_n(x_0, x_1, \cdots , x_n) = F_{n+1}(x_0, x_1, \cdots , x_n, \infty),$ (1.5)

for all $-\infty < x_i < \infty$, and $i = 0, 1, \cdots , n, n \geq 0$. A consistent family of finite dimensional joint cdfs thoroughly describes a discrete time stochastic process, meaning that any probabilistic query regarding $\{X_n, n \geq 0\}$ may be response about terms regarding $\{F_n, n \geq 0\}$. To establish a probability space (Ω, F, P) on which the process is described is to "fully describe" a stochastic process, technically speaking. For further information on stochastic processes, the more inquisitive reader is directed to more sophisticated works on the subject.

In continuation, we use the case as $T = [0, \infty)$. Since this scenario involves an infinite number of random variables, completely characterising a continuous time stochastic process, $\{X(t), t \geq 0\}$, is unfortunately not as easy as it might seem. If we can assume certain things about the consistency of the sample pathways of the process, the issue can be made simpler. We won't get into details here; instead, we'll state the key finding:

Assume that the sample pathways of $\{X(t), t \geq 0\}$, are right continuous with left limits, with probability 1, that is

$\lim_{s\downarrow t} X(s) = X(t),$ (1.6)

and for any t, $\lim_{s\uparrow t} X(s)$ exists. Consider, as well, that the sample pathways have a finite number of discontinuities with probability 1 over a finite period of time. So a consistent family of finite dimensional joint cdfs can fully represent $\{X(t), t \geq 0\}$

$F_{t_1,t_2,\cdots,t_n}(x_1, x_2, \cdots , x_n) = P(X(t_1) \leq x_1, X(t_2) \leq x_2, \cdots , X(t_n) \leq x_n),$ (1.7)

for all $-\infty < x_i < \infty$, $i = 1, \cdots , n, n \geq 1$ and all $0 \leq t_1 < t_2 < \cdots < t_n$.

Independent and identically distributed random variables are one example. Let $\{X_n, n \geq 1\}$ be a series of independently distributed random variables that share the same distribution $F(\cdot)$. We can construct the following stable family of joint cdfs, demonstrating that $F(\cdot)$ completely captures the behaviour of this stochastic process:

$F_n(x_1, x_2, \cdots , x_n) = \prod_{i=1}^{n} F(x_i),$ (1.8)

for all $-\infty < x_i < \infty$, $i = 1, 2, \cdots , n$, and $n \geq 1$.

Although it lacks any intriguing structure, a sequence of random variables is the most basic form of stochastic process. However, as demonstrated in the following example, one can build more intricate and fascinating stochastic processes from it.

Example 2:- Considering random walk; assume $\{X_n, n \geq 1\}$ be like above Example 1.

Define $S_0 = 0, S_n = X_1 + \cdots + X_n, n \geq 1.$ (1.9)

A random walk is the stochastic process $\{S_n, n \geq 0\}$. Since the joint distribution of $(S_0, S_1, \cdots , S_n)$ is entirely dictated by that of $(X_1, X_2, \cdots , X_n)$, which, as seen in Example 1, is determined by $F(\cdot)$, it is also entirely characterised by $F(\cdot)$.

Conclusion

A rectangular array of numbers is known as a matrix in mathematics (plural matrices, or less frequently, matrixes), as depicted to the right. The term "tensor" is used to describe higher-dimensional, such as three-dimensional, arrays of numbers as opposed to higher-dimensional matrices with only one column or row. Matrices can be multiplied according to a rule that corresponds to the composition of linear transformations, as well as added and subtracted entrywise. Except for the fact that matrix multiplication is not commutative and can fail, these operations meet the common identities. The representation of linear transformations-higher-dimensional analogues of the linear functions of the type $f(x) = cx$, where c is a constant - is one application of matrices. The coefficients of a set of linear equations can also be tracked by matrices. For a square matrix, the behaviour of solutions to the corresponding system of linear equations is governed by the determinant and inverse matrix (where it exists), and the associated linear transformation's geometry is revealed by the eigenvalues and eigenvectors. There are various uses for matrices. They are used in many areas of physics, such as matrix mechanics and geometrical optics. Further research into matrices with an endless number of rows and columns resulted from the latter. In graph theory, matrices are used to record the distances between knot sites, such as cities connected by roads, and in computer graphics, matrices are used to encode the projections of three-dimensional space onto a two-dimensional screen. Matrix calculus applies traditional analytical concepts like function derivatives and exponentials to matrices. In order to solve ordinary differential equations, the latter is frequently required. A square mathematical matrix is used by the 20th-century musical movements serialism and dodecaphonism to determine the structure of musical intervals. Due to their ubiquitous application, a lot of work has been spent into developing effective matrix computing techniques, especially if the matrices are large. To do this, there exist a variety of matrix decomposition techniques that describe matrices as the products of other matrices with specific features, thereby theoretically and practically simplifying computations. Sparse matrices, or matrices with a large percentage of zeros, are common in the finite element method simulation of mechanical experiments and frequently enable more specialised methods to do these jobs. The former is a crucial idea in linear algebra due to the strong connection between matrices and linear transformations. Rings or other entry types, such as those in more general mathematical areas, are also used.

References

1. M.F. Pellissetti and R.G. Ghanem, Iterative solution of systems of linear equations arising in the context of stochastic finite elements, Advances in Engineering Software, 31(8-9): 607-616, 2000.
2. C.F. Li, Y.T. Feng and D.R.J. Owen, Explicit solution to the stochastic system of linear algebraic equations $(\alpha_1 A_1 + \alpha_2 A_2 + \cdots + \alpha_m A_m)$ $x = b$, *Computer Methods in Applied Mechanics and Engineering, 195(44-47): 6560-6576, 2006.*
3. Yadav, Ankit and Dwivedi, Dr. P.K., “Model Based on the Utilization of Utilitarian Documentation”, Cosmos An International Journal of Management, 5(2): 1-3, 2016.
4. Kumar Puneet, “Relationship between binary tree and Boolean algebra changing the parameter of the Boolean expression: A Functional Approach”, Bulletin of Pure and Applied Sciences, 23E(1): 209-213, 2004.
5. Kumar Puneet, “Measure the number of circuits by increment the parameter in Boolean algebra”, Bulletin of Pure and Applied Sciences, 24E(2): 423-429, 2005.
6. Rani, Minaxi, “A Remark on the Square Root of a Complex Binomial Quadratic Surd”, Globus An International Journal of Management & IT, 6(1): 15-17, 2014.
7. Kumar Puneet, “A Comparative Study of Information System’s Security by using Graphs”, Enterprise Information Systems & Technology, MacMillan India Ltd., pp 222-227, ISBN 0230-63516-4, 2008.
8. Aghi, Manish and Dwivedi, Dr. P.K., “A Study on Animal Eating and Nonlinear Advancement”, Cosmos Journal of Engineering & Technology, 7(1): 1-3, 2017.
9. Annareddy, Maheswarareddy and Saini, Dr. K.K., “A Study on Cross Layer Intrusion Detection Systems”, Cosmos Journal of Engineering & Technology, 7(1): 1-3, 2017.
10. Trivedi, Dr. Sunil Kumar and Babu, Dr. Yogendra, “Role of Science and Technology in the Present Scenario”, Globus An International Journal of Management & IT, 9 (1): 1-3, 2017.
11. Mahapatra, Cosmena, “The Discussion on Secure Routine Protocols”, Cosmos Journal of Engineering & Technology, 4(1): 1-3, 2014.
12. Devi, Monika, “A Study on Fuzzy Generalized in Continuous Mapping”, Cosmos Journal of Engineering & Technology, 4(1): 4-6, 2014.
13. Yadav, Ankit and Dwivedi, Dr. P.K., “Model Based on the Utilization of Utilitarian Documentation”, Cosmos An International Journal of Management, 5(2): 1-3, 2016.
14. Kumar Puneet, “Relationship between binary tree and Boolean algebra changing the parameter of the Boolean expression: A Functional Approach”, Bulletin of Pure and Applied Sciences, 23E(1): 209-213, 2004.
15. Kumar Puneet, “Measure the number of circuits by increment the parameter in Boolean algebra”, Bulletin of Pure and Applied Sciences, 24E 2): 423-429, 2005.
16. Rani, Minaxi, “A Remark on the Square Root of a Complex Binomial Quadratic Surd”, Globus An International Journal of Management & IT, 6(1): 15-17, 2014.
17. Kumar Puneet, “A Comparative Study of Information System’s Security by using Graphs”, Enterprise Information Systems & Technology, MacMillan India Ltd., pp 222-227, ISBN 0230-63516-4, 2008.
18. Aghi, Manish and Dwivedi, Dr. P.K., “A Study on Animal Eating and Nonlinear Advancement”, Cosmos Journal of Engineering & Technology, 7(1): 1-3, 2017.

19. Annareddy, Maheswarareddy and Saini, Dr. K.K., “A Study on Cross Layer Intrusion Detection Systems”, Cosmos Journal of Engineering & Technology, 7(1): 1-3, 2017.
20. Trivedi, Dr. Sunil Kumar and Babu, Dr. Yogendra, “Role of Science and Technology in the Present Scenario”, Globus An International Journal of Management & IT, 9 (1): 1-3, 2017.