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A STUDY ON FUZZY GENERALIZED IN CONTINUOUS MAPPING

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Abstract

In this paper we introduce the concept of fuzzy generalized closed sets. Fuzzy generalized continuous mapping. Fuzzy generalize irresolute maps. Fuzzy generalized homeomorphism, strongly fuzzy generalized continuous; perfectly fuzzy generalized continuous maps.

Keywords: *Fuzzy generalized closed sets, Fuzzy generalized closed set, Fuzzy generalized, Continues fuzzy generalized connections.*

Introduction

The concept of generalized closed sets in a topological space was introduced by Veera Kumar and it was used by different authors to define several topology properties. Nevine introduced generalized closed sets in topological spaces. The concept of fuzzy closed is an important role in fuzzy topological spaces. Along this line many weaker forms of fuzzy continuity and perfect forms of fuzzy continuity were introduced. In this paper we introduced and study on fuzzy generalized extremely disconnectedness. In section 2 is devoted to fuzzy generalized closed sets and their properties. In section 3 the concept of fuzzy generalized continuous mapping and their properties are studied. Throughout this paper X and Y means fuzzy topological spaces in Change. For a fuzzy set A of a topological spaces X, the notations Cl (A), lnt (A) and I - A will respectively stand for the fuzzy closure, fuzzy interior and compliment of A. By 0 and l we will mean the fuzzy sets with constant membership function 0 (zero function) and 1 {unit function) respectively.

Definition 1

Let X be a fuzzy topological space. A fuzzy set it in X is called fuzzy generalized closed (in short fgc) if Cl (λ) μ. Whenever λ ≤ μ and μ is fuzzy open.

Definition 2

Let X be a fuzzy topological space. A fuzzy set p in X is called fuzzy generalized closed (in short fgc) if Cl (p) ≤ q, whenever p≤ q and q is fuzzy generalized open in X.

Theorem

Every fuzzy generalized closed set is a fuzzy generalized closed set. The converse of above theorem need not be true.

Example

Let X={a, b, c}. Define A, B, C: X ~> [0, l] as follows

A (a) = l A (b) = 0 A (c) =0
B (a) = 0 B (b) = l B (c) =l
C (a) = 0 C (b) = 0 C (c) =l

Consider the fuzzy topology T = {0, l, C}. It is clear that A, B is fuzzy generalized closed set. But A, B is not fg*-closed set.

Theorem

If A and B are fg*-closed set in X, then Av B is a fg*-closed set in X.

Proof:

Assume that A and B are fg*-closed set in X. Let q be a fuzzy generalized open set such that Av B ≤q. Then A ≤ q and B ≤ q. Therefore by definition 2.2. Cl (A) ≤ q and Cl (B) ≤ q. Therefore Cl (A v B) = Cl (A) v Cl (B) ≤ q v q = q. Hence A∨B is fg* closed set in X,

Theorem

Let A is a fg*-closed set in fuzzy topological space X, and A ≤ B ≤ Cl (A), the: C is fg*-closed set in X.

Proof:

Let q be a fuzzy generalized open set such that B ≤ q. Then A ≤ q, since A is fax; generalized* closed set in X, Cl (A) ≤ q. Now B ≤ Cl (A) implies Cl (B) ≤ Cl (A) ≤ q. Hence E fuzzy generalized* closed set in X. .

Theorem

Let X be a fuzzy topological space. A fuzzy set A of X is fgi open iff, B ≤ lnt (A) whenever B is fg-closed set and B ≤ A.

Proof:

Let A be a fg*-open set and B is fg-closed such that B ≤ A implies l-B 2 l-A and l-A fg*-closed. S0, Cl (l-A) ≤ l-B implies (l-Cl (l-A)) .2 (l-(l-B)) = B. But (l-Cl (1-A)) = Int (A) / [≤]. Thus B ≤ lnt (A). Conversely, suppose that A is fuzzy set such that B ≤ lnt (A),

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whenever B is fg-closed and B ≤ A. We show that l-A is fg*-closed set. Let l-A ≤ B, where B is fg-open. Since l-A ≤ B implies that l-B ≤ A.

By assumption that we must have l-B ≤ lnt (A) or l-lnt (A) ≤ B. Now l-Int: (A) = Cl (l-A) [3] which implies that Cl (1-A) ≤ B and l-A is fg*-closed set.

Theorem
Let A is a fg*-open set in topological space X and lnt (A) ≤ B ≤ A, then B is fg-open in X.

Proof:
Given that lnt (A) ≤ B ≤ A, we have l-A ≤ l-B ≤ l-lnt (A). Since A is fg*-open in .1".

Therefore l-A is fg*-closed in X and so it follows by Theorem 2.6 that I-B is fg*-closed in P-I Hence B is fg*-open in X.

Theorem
Let X be a fuzzy topological space and fg-open stand for the family of all fuzz; generalized open set of X and fg-closed stand for the family of all fuzzy generalized closed subset of X.

If every fuzzy subset of X is a fg*-closed set then fg-open (X) = fg=closed (X). **(1)**

Proof:
Let us assume that every p ϵ q is fg*-closed set in X. Let p is fg-open (X). Since p≤p and p is fg*-closed, we have Cl (p) ≤ p, but ≤ p Cl (p). Therefore Cl (p) = p fg-closed (X) implies fg-open (X) ≤ FG-Closed (X).

Assume that p is fg-closed (X) then l-p is fg-open (X) 5 fg-closed (X) implies l-p is fg-closed (X) implies p ϵ fg-open (X). Hence fg-closed (X) ≤ fg-open (X) From (l) and (2) gives fg-open (X) = fg-closed (X).

Remark
A and B are fg*-closed set, but $A_\Lambda B$ is not fg*-closed in X.

Example
Let X = {a, b}. Define A, B, C: X → [0,1] as follows
A (a) = 0.9 A (b) = 0
B (a) = 0.3 B (b) = l
C (a) = 0.6 C (b) = 0

Consider the fuzzy topology I = {0, 1, C} it is clear that A and B are fg*-closed set. But A/\B is not fg* closed.

Fuzzy Generalized Continuous Mapping and Their Properties

Definition 3
A map f: X → Y is called generalized fuzzy continuous if the inverse image of every fuzzy closed set in Y is fg-closed in X.

Definition 4
A map f: X → Y is called fuzzy generalized* continuous if the inverse image of every fuzzy closed set in Y is fg*-closed set in X.

Theorem
If f: X → Y is fuzzy continuous then it is" fg*-continuous.

Proof:
Assume that f: X → Y is fuzzy continuous let β be any fuzzy closed set in Y, since f: X → Y is fuzzy continuous. Then f "(β) is fuzzy closed in X and hence f "(β) is fg*-closed set in X. Hence f is fg*-continuous. The converse of above theorem need not be true

Example
Let X = {a, b, c} and Y = {r, s}. Define τ = {0, l, A} where A: X → [0,1] is such that A (a) = 0, A (b) = O, A (c) = l and σ = {0, l, B} where B: Y → [0,l] is such that B (r) =0, B (s) = l. Also define f: X → Y as f(a) = s. f(b) = s, f(c) = r. Then f is fg*-continuous, but not fuzzy continuous. '

Theorem
Let f: X → Y be a map. Then the following are equivalent
(a) f is fg*continuous.
(b) The inverse image of each fuzzy open set in Y is fg*~open in X.

Proof:
(a) → (b) Let r be a fuzzy open set in Y. Then r^s is fuzzy closed set in Y. By (a) $f^{-1}(r^c)$ is fuzzy closed in X. $f^1(r^c)(X) = r^c(f(x)) = (1- r)(f(x)) = 1 - f^{-1}(r$ (X)) implies, $f^1(r^c) =$ 1- f $-^1$) (r) Since f is fg*-continuous $f^{-1}(r^c) = (X) = r^c = 1- f^1(r)$ isfg*-closed in X. Therefore f^1 (r) is fg*-open in X.

(b) : ⇒ (a) Let s be a fuzzy closed set in Y. Then S^c is fuzzy open in By (b) f^1 (s Â°) is fgi* -open in X. As we have already seen in (a) ⇒ (b), $f^{-1}(S^c)$ = 1- f $-^1$ (S). Therefore f $-^1$ (S) is fg*--closed i" X. Thus f is fg*-continuous.

Now we define and introduce the notation of the generalized* fuzzy closure operator (in short g*Cl

(A)) for any fuzzy' set A in X as follows g*Cl (A) = Λ {x : A ≤ x, and x is fg*-closed}.

Theorem

Let f: X →Y be fg*-continuous. Then if [g*Cl) (A)] ≤ Cl (f (A)), where A is an fuzzy set in X.

Proof:

Since Cl (f (A)) is fuzzy closed set in Y and f is f* continuous f^{-1} (Cl (f (A)) (Ci (f(A))) is fg*closed in X, A ≤ f^{1} (Cl (f (A))) and so g*Cl (A) 5 $f\text{-}^{1}$Cl (f(A)}}, therefore f(g*Cl (A)) ≤ Cl (f(A)).

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