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THE DISCUSSION ON CHARACTERIZATION OF GENERALIZED INFORMATION MEASURES

Sachin Doley[1]

Abstract

The present paper explains the four-color problem in one of his lectures in 1840. About 10 years later, A. De Morgan (1806-1871) discussed this problem with his fellow mathematicians in London. Dc Morgan's letter is the first authenticated reference to the four-color problem. The problem became well known after Cayley published it in 1879 in the first volume of the Proceedings of the Royal Geographic Society. To this day, the four-color conjecture is by far the most famous unsolved problem in graph theory; it has stimulated an enormous amount of research in the field.

Keywords: Information Measures, Characteristics.

Introduction

Consider two probability distributions $P = (p_1, p_2, \ldots, p_n) \in \Delta_n$ and $Q = (q_1, q_2, \ldots, q_n) \in \Delta_n$ associated with a discrete random variable X taking finite number of values $\{x_1, x_2, \ldots, x_n\}$ where

$$\Delta_n = \left\{ P = (p_1, p_2, \ldots\ldots, p_n) : p_i \geq 0, \sum_{i=1}^{n} p_i = 1 \right\}.$$

Consider a generalized measure of information between two distributions P and Q given by

$$I_a^c(P; Q) = \left(2^{(a-1)c} - 1\right)^{-1} \left\{ \left[\sum_{i=1}^{n} p_i^a q_i^{b-a} \right] - 1 \right\}, \ldots.. (1)$$

Where $a > 0$, $b > 0$, $a \neq b$, $c \neq 0$ are arbitrary parameters.

The following are the particular cases of the measure (1):

(i)

$$\lim_{c \to 0} I_{a,b}^c(P; Q) = I_{a,b}^c(P; Q) = \frac{1}{a-b} \log_2 \left(\sum_{i=1}^{n} p_i^a q_i^{b-a} \right), \quad a \neq b, a, b > 0 \qquad (2)$$

For b = 1,

$$I_{a,b}^c(P; Q) = I_{a,1}^c(P; Q) = \frac{1}{a-b} \log_2 \left(\sum_{i=1}^{n} p_i^a q_i^{1-a} \right) \quad a \neq 1, a, b > 0 \qquad (3)$$

Which is relative information of order a

For a = 1,

$$I_{a,b}^0(P; Q) = I_{1,b}^0(P; Q) = \frac{1}{a-b} \log_2 \left(\sum_{i=1}^{n} p_i \ q_i^{b-1} \right) \quad b \neq 1, b > 0 \qquad (4)$$

Which is an inaccuracy of order b.

(ii) When c = 1,

$$I_{a,b}^1(P; Q) = (2^{a-b} - 1)^{-1} \left\{ \left[\sum_{i=1}^{n} p_i^a q_i^{b-a} \right] \right\}, a, b > 0, a \neq b \ (5)$$

Which is a generalized measure of relative information and inaccuracy.

For b = 1,

$$I_{a,1}^1(P; Q) = (2^{a-b} - 1)^{-1} \left\{ \left[\sum_{i=1}^{n} p_i^a q_i^{1-a} - 1 \right] \right\}, a > 0, a \neq 1 \ (6)$$

Which is a relative information of degree a

For a = 1,

$$I_{1,b}^1(P; Q) = (2^{1-b} - 1)^{-1} \left\{ \left[\sum_{i=1}^{n} p_i^a q_i^{b-1} \right] \right\}, b > 0, b \neq 1 \ (7)$$

Which is an inaccuracy of degree b

(iii)

$$\lim_{a \to 1} I_{a,1}^0(P; Q) = \lim_{a \to 1} I_{a,1}^1(P; Q) = \sum_{i=1}^{n} p_i \ \log_2(p_i / q_i) \quad (8)$$

Which is a relative information, directed divergence or cross-entropy between the distributions P and Q

(iv)

$$\lim_{b \to 1} I_{1,b}^0(P; Q) = \lim_{b \to 1} I_{1,b}^1(P; Q) = -\sum_{i=1}^{n} p_i \ \log_2(q_i) \quad (9)$$

Which is an inaccuracy measure between the distribution P and Q?

Various authors have paid attention in characterizing these measures of information, for brief review refer to Mathai and Rathie, and Taneja.

[1]*Research Scholar, Sunrise University, Alwar, Rajasthan*

The measure (1) for b =1, c = $\frac{d-1}{a-1}$ has been characterized by Sharma and Mittal and for a = 1, c = $\frac{d-1}{b-1}$ has been characterized by Sharma and Gupta under mean value considerations. The aim of this study is to give an axiomatic characterization of the measure (1) under simpler approach. The extensions of (1) to three and more probability distributions are also specified. Whereas, the characterization of measure (1) involving only one probability distribution is given by the author.

Characterization

In this chapter we shall characterize the measure (I) under certain axioms. This characterization is given in the following theorem:

Theorem

Let $F : \Delta_n^2 \rightarrow \mathcal{R}$ *(reals)*, $n \geqslant 2$ be a function satisfying the following axioms:

(1) F(P; Q) = $k_1 \left[\sum_{i=1}^{n} f(p_i, q_i)\right]^c + k_2$, k_1, k_2, c $\neq$ 0, where f is a real continuous function defined over $[0, 1]^2$; k_1, k_2 are arbitrary constants and c is an arbitrary parameter.

(2) $F(P*U; Q*V) = F(P; Q) + F(U; V) - \frac{1}{k_2} F(P; Q)F(U; V)$, for all $P, Q \in \Delta_n$, $U, V \in \Delta_m$, and $P*U, Q*V \in \Delta_{nm}$.

(3) $F(1, 0; \frac{1}{2}, \frac{1}{2}) = 1$. Then $F(P; Q) = I^c_{a,b}(P; Q$

Proof. From axioms (I) and (II), we have

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} f(p_i u_j, q_i v_j)\right]^c = \left(-\frac{k_1}{k_2}\right)^{1/c} \left[\sum_{i=1}^{n} f(p_i q_i)\right]^c . \left[\sum_{i=1}^{n} f(u_j v_j)\right]^c,$$

i.e.

$$\sum_{i=1}^{n}\sum_{j=1}^{m} f(p_i u_j, q_i v_j) = \left(-\frac{k_1}{k_2}\right)^{1/c} \sum_{i=1}^{n}\sum_{j=1}^{m} f(p_i q_i) f(u_j v_j) \quad (10)$$

It is easy to see that the functional equation (10) is equivalent to the following:

$$f(pu, qv) = \left(-\frac{k_1}{k_2}\right)^{\frac{1}{c}} f(p,q) f(u,v) \quad (11)$$

for all reals p. q, $u, v \in [0, 1]$.

Now substituting

$$\left(-\frac{k_1}{k_2}\right)^{1/c} f(p,q) = \phi(p,q). \quad (12)$$

in (11), we get

$$\phi(pu, qv) = \phi(p,q).\phi(u,v) \quad (13)$$

The most general (non-trivial) continuous solutions of the functional equation (13) are given by

$$\phi(p,q) = p^\alpha q^\beta, \alpha > 0, \beta \neq 0. \quad (14)$$

For simplicity write α = a and β = b — a. Then

$$\phi(p,q) = p^a q^{b-a}, a > 0, b \neq a. \quad (15)$$

Finally, axioms (I) and axiom (11), (12) and (15) together give the required result.

Review of Literature

About the time of Kirchhoff and Cayley, two other milestones in graph theory were laid. One was the four-color conjecture, which states that four colors

are sufficient for coloring any atlas (a map on a plane) such that the countries with common boundaries have different colors.

It is believed that A. F. Mobius (1790-1868) first presented the four-color problem in one of his lectures in 1840. About 10 years later, A. De Morgan (1806-1871) discussed this problem with his fellow mathematicians in London. Dc Morgan's letter is the first authenticated reference to the four-color problem. The problem became well known after Cayley published it in 1879 in the first volume of the Proceedings of the Royal Geographic Society. To this day, the four-color conjecture is by far the most famous unsolved problem in graph theory; it has stimulated an enormous amount of research in the field.

The other milestone is due to Sir W. R. Hamilton (1805-1865). In the year 1859 he invented a puzzle and sold it for 25 guineas to a game manufacturer in Dublin. The puzzle consisted of a wooden, regular dodecahedron (a polyhedron with 12 faces and 20 corners, each face being a regular pentagon and three edges meeting at each corner; see Fig.). The corners were marked with the names of 20 important cities: London, New York, Delhi, Paris, and so on. The object in the puzzle was to find a route along the edges of the dodecahedron, passing through each of the 20 cities exactly once.

Although the solution of this specific problem is easy to obtain, to date no one has found a necessary and sufficient condition for the existence of such a route (called Hamiltonian circuit) in an arbitrary graph.

This fertile period was followed by half a century of relative inactivity. Then a resurgence of interest in graphs started during the I 920s. One of the pioneers in this period was D. Konig. He organized the work of other mathematicians and his own and wrote the first book on the subject, which was published in 193.6.

The past 30 years has been a period of intense activity in graph theory - both pure and applied. A great deal of research has been done and is being done in this area. Thousands of papers have been published and more than a dozen books written during the past decade. Among the current leaders in the field are Claude Berge, Oystein Ore (recently deceased), Paul Erdös, William Tutte, and Frank Harary.

The researcher has made an attempt to review the literature related to finding shortest path in various applications. However there are number of literatures.

List of other study title in journals & articles related to research problem:

Dr. Natarajan Meghanathan reviewed Dijkstra algorithm and Bellman-Ford algorithm for finding shortest path in graph. He conclude that the time complexity for Dijkstra algorithm is O(|E|*log|V|) and the time complexity of Bellman-Ford algorithm is O(|V||E|).

Lili Cao, Xiaohan Zhao, Haitao Zheng, and Ben Y. Zhao conclude that search for shortest paths is an essential primitive for a variety of graph-based applications, particularly those on online social networks. For example, LinkedIn users perform queries to find the shortest path "social links" connecting them to a particular user to facilitate introductions. This type of graph query is challenging for moderately sized graphs, but becomes computationally intractable for graphs underlying today's social networks, most of which contain millions of nodes and billions of edges. They propose Atlas, a novel approach to scalable approximate shortest paths between graph nodes using a collection of spanning trees. Spanning trees are easy to generate, compact relative to original graphs, and can be distributed across machines to parallelize queries. They demonstrate its scalability and effectiveness using 6 large social graphs from Facebook, Orkut and Renren, the largest of which includes 43 million nodes and 1 billion edges. They describe techniques to incrementally update Atlas as social graphs change over time. They capture graph dynamics using 35 daily snapshots of a Facebook network, and show that Atlas can amortize the cost of tree updates over time. Finally, they apply Atlas to several graph applications, and show that they produce results that closely approximate ideal results.

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