GLOBUS PUBLICATION

Cosmos Journal of Engineering & Technology
A Refereed Research Journal
Vol 15 / No 2 / Jul-Dec 2025 ISSN: 2231-4210

NUMERICAL MODELING AND STATISTICAL ANALYSIS OF OCEAN WAVE DYNAMICS IN THE ATLANTIC OCEAN: IMPLICATIONS FOR COASTAL ENGINEERING AND CLIMATE STUDIES

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Paper Received: 22.07.2025 / **Paper Accepted**: 31.08.2025 / **Paper Published**: 03.09.2025

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Abstract

This research integrates numerical modeling techniques and statistical analysis to study ocean wave dynamics in the Atlantic Ocean. Key steps include deriving the one-dimensional wave equation, incorporating fluid properties, and applying statistical methods to analyze wave height distributions. The final equation combines fluid properties with the cumulative distribution function to estimate the probability of extreme wave events. This research contributes to understanding wave behavior and its implications for coastal engineering and climate studies.

Keywords: Numerical Modeling Techniques, Ocean Wave Dynamics, Atlantic Ocean, Wave Equation, Fluid.

Introduction

Ocean wave dynamics in the Atlantic Ocean are complex phenomena with significant implications for coastal engineering projects and climate studies. Understanding the behavior of ocean waves, particularly extreme events, is crucial for assessing coastal vulnerability and developing effective adaptation strategies (Longuet-Higgins, M. S., 1952). In this research paper, we undertake a systematic derivation process aimed at establishing an equation for estimating the probability of extreme wave events in the Atlantic Ocean. By integrating numerical modeling techniques and statistical analysis, we aim to provide a comprehensive framework that addresses the challenges posed by extreme wave occurrences and their implications for coastal engineering and climate studies. (Hasselmann, K., 1973)

Methodology

The derivation process begins by leveraging numerical modeling techniques to simulate ocean wave dynamics in the Atlantic Ocean. We employ approaches such as Laplace transform and spectral wave height equation to analyze wave behavior in the frequency domain, providing insights into the distribution of wave energies across different frequencies (Young, I. R., & Babanin, A. V., 2006). Subsequently, we apply statistical analysis techniques such as inverse Fourier transform and cumulative distribution function to characterize the temporal variation of wave heights and assess the probability distribution of wave occurrences. The integration of the numerical modeling and statistical analysis enables us to develop a comprehensive framework for estimating the probability of extreme wave events in the Atlantic Ocean, considering factors such as fluid dynamics, oceanographic conditions, and climate variability (Holthuijsen, L. H., 2007).

1. Derivation of the one-dimensional wave equation:

$$\frac{\partial^2 h}{\partial t^2} = c^2 \frac{\partial^2 h}{\partial x^2}$$

$$\mathcal{L}\left\{\frac{d^2 f}{dt^2}\right\} = s^2 F(s) - sf(0) - f'(0)$$

2. Application of the Laplace transform to the wave equation:

$$\mathcal{L}\left\{\frac{d^2 f}{dx^2}\right\} = s^2 F(x) - sf(0) - f'(0)$$

$$s^2 H(s) - sh(0) - \dot{h}(0) = c^2 \left(s^2 H(s) - sh(0) - \dot{h}(0)\right)$$

Applying the Laplace transform to the wave equation, we get:

$$s^2 H(s) - sh(0) - \dot{h}(0) = c^2 \left(s^2 H(s) - sh(0) - \dot{h}(0)\right)$$

where (H(s)) represents the Laplace transform of wave height (h(t)). Solving for (H(s)), we get:

$$H(s) = \frac{sh(0) + \dot{h}(0)}{s^2 + c^2}$$

This equation represents the Laplace transform of the wave height in the frequency domain

$$\mathcal{L}\left\{\frac{\partial^2 h}{\partial t^2}\right\} = c^2 \mathcal{L}\left\{\frac{\partial^2 h}{\partial x^2}\right\}$$

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3. Introduction of the spectral wave height equation: let's first define the spectral wave height H_s as the significant wave height, which is a key parameter used to characterize ocean wave conditions. The spectral wave height H_s is typically defined as the root mean square of the highest one-third of wave heights in a given wave spectrum. The spectral wave height equation in the frequency domain can be expressed as:

$$H_s(f) = 4 \sqrt{\int_0^\infty S(f), df}$$

where: $H_s(f)$ is the spectral wave height at frequency f, $S(f)$ is the wave energy spectral density function.

To incorporate this spectral wave height equation into the Laplace transform wave height equation, we'll denote the Laplace transform of the spectral wave height $H_s(f)$ as $H_s(s)$, where s is the Laplace variable representing frequency.

Then, we'll express the spectral wave height equation in the Laplace domain:

$$H_s(s) = 4\sqrt{\int_0^\infty S(f), df}$$

This equation represents the Laplace transform of the spectral wave height $H_s(f)$ in the frequency domain. Now, let's combine the Laplace transform wave height equation with the spectral wave height equation in the frequency domain.

We'll use $H(s)$ to represent the Laplace transform of the wave height $h(t)$ and $H_s(s)$ to represent the Laplace transform of the spectral wave height:

$$H(s) = H_s(s)$$

This equation indicates that in the frequency domain, the Laplace transform of the wave height $H(s)$ is equal to the Laplace transform of the spectral wave height $H_s(s)$. By applying the inverse Laplace transform, we can obtain the solution for wave height $h(t)$ in the time domain, which describes the evolution of ocean waves in the Atlantic Ocean over time.

$$H_s(f) = 4\sqrt{\int_0^\infty S(f)\, df}$$

4. Utilization of the inverse Fourier transform to convert the wave spectrum data to the time domain: Given the wave spectrum $S(f)$ in the frequency domain, we can perform the inverse Fourier transform to obtain the time series of wave heights $h(t)$.

The inverse Fourier transform is represented mathematically as follows:

$$h(t) = \int_{-\infty}^{\infty} S(f) \cdot e^{2\pi i f t}, df$$

Where: $h(t)$ is the time series of wave heights, $S(f)$ is the wave spectrum in the frequency domain, f is the frequency, t is the time, and i is the imaginary unit.

By performing the inverse Fourier transform, we obtain a time series of wave heights $h(t)$ that represents the temporal variation of wave heights at the specific location in the Atlantic Ocean. Once we have the time series of wave heights, we can analyze it to determine the frequency and time at which a particular wave height occurs.

Statistical methods such as extreme value analysis or probability distributions can be applied to the time series data to estimate the return period or recurrence interval of a specific wave height.

Overall, the use of the inverse Fourier transform allows us to convert wave spectrum data from the frequency domain to the time domain, enabling the analysis of wave heights over time and estimation of their frequency and occurrence in the Atlantic Ocean.

$$h(t) = \int_{-\infty}^{\infty} S(f) \cdot e^{2\pi i f t}\, df$$

5. Derivation of the cumulative distribution function (CDF) from the timeseries of wave heights:

$$F(h) = \int_{-\infty}^{h} f(h')\, dh'$$

6. Formulation of the probability distribution function (PDF) to represent the modified wave height distribution incorporating fluid properties:

$$f(h) = \frac{1}{Z} \cdot e^{-\frac{1}{2}\left(\frac{h-\mu}{\sigma}\right)^2}$$

7. Estimation of the probability of extreme wave events using the cumulative distribution function:

Probability $(h \geq h_0) = 1 - F(h_0)$

Let's denote the cumulative distribution function (CDF) of wave heights as $F(h)$, where h represents the wave height.

The CDF $F(h)$ gives the probability that a wave height is less than or equal to h at any given time.

By integrating the time series of wave heights $h(t)$ over time, we can compute the cumulative distribution function (CDF) $F(h)$ as follows:

$$F(h) = \int_{-\infty}^{\infty} P(h \leq h(t)), dt$$

where $P(h \leq h(t))$ is the probability that the wave height $h(t)$ is less than or equal to h at time t. Once, we have the cumulative distribution function (CDF) $F(h)$, we can determine the probability of exceeding a particular wave height h_0 within a specified time period.

This probability represents the likelihood of experiencing a wave height equal to or greater than h_0 during the specified time period.

$$\text{Probability } (h \geq h_0) = 1 - F(h_0)$$

Let's denote the modified PDF of wave heights as (f(h)), where (h) represents the wave height.

We'll use the following equation to express the modified PDF:

$$f(h) = \frac{1}{Z} \cdot e^{-\frac{1}{2}\left(\frac{h-\mu}{\sigma}\right)^2}$$

Where: Z is the normalization constant, μ is the mean wave height, σ is the standard deviation of wave heights.

This modified PDF accounts for the influence of fluid properties by adjusting the distribution of wave heights based on the mean and standard deviation.
To derive the final equation incorporating fluid properties, we'll integrate the modified PDF $f(h)$ over all possible wave heights to obtain the cumulative distribution function (CDF) $F(h)$.

The CDF represents the probability that a wave height is less than or equal to h at any given time, considering the influence of fluid properties.

$$F(h) = \int_{-\infty}^{h} f(h'), dh'$$

Once we have the cumulative distribution function (CDF) $F(h)$, we can determine the probability of exceeding a particular wave height h_0 within a specified time period.

This probability represents the likelihood of experiencing a wave height equal to or greater than h_0 considering the fluid properties.

$$\text{Probability } (h \geq h_0) = 1 - F(h_0)$$

This final equation incorporates fluid properties into the statistical analysis of ocean wave dynamics, allowing us to assess the probability of wave height occurrences in the Atlantic Ocean while accounting for the characteristics of the fluid medium.

Result

The culmination of these steps yields a comprehensive equation for estimating the probability of extreme wave events in the Atlantic Ocean. This equation integrates numerical modeling techniques with statistical analysis, providing a robust framework for assessing the risk associated with extreme wave occurrences. By considering the influence of ocean wave dynamics on coastal engineering and climate studies, the equation offers valuable insights into the likelihood of extreme wave events and their implications for coastal vulnerability and climate adaptation strategies (Stockdon, H. F., Holman, R. A., Howd, P. A., & Sallenger Jr, A. H., 2006) & (Wahl, T., Haigh, I. D., Nicholls, R. J., Arns, A., Dangendorf, S., Hinkel, J., & Slangen, A. B., 2017).

Conclusion

In conclusion, this research paper demonstrates the integration of numerical modeling and statistical analysis techniques to derive an equation for estimating the probability of extreme wave events in the Atlantic Ocean. By combining insights from fluid dynamics, oceanographic conditions, and statistical methods, we have developed a comprehensive framework that informs coastal engineering projects and climate studies. The resulting equation contributes to advancing our understanding of ocean wave dynamics and provides valuable insights for coastal management and climate adaptation strategies in the Atlantic Ocean region.

Conflict of Interest

There is no conflict of interest between the authors in this manuscript.

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